1. Introduction

Field-performance data of machine parts formed by finish machining essentially depend on the state and properties of the surface and subsurface layers of material. The correct methods of finishing and treatment modes of these methods ensure the design requirements of the machined parts and has a decisive influence on their quality and attractive appearance.

Abrasive blasting is one of the methods that can be applied to the machine parts for finishing treatment. This method provides matte surface, removes visible signs of treating by previous instrument (scratches, wire-edges), increases fatigue strength and wear resistance of the machine parts and in consequence influences forming compression stress on subsurface layers.

New promising application for abrasive blasting can be endoprosthesis manufacturing, that is rapidly developed at present. Using methods of the abrasive blasting for endoprosthesis fastening parts can allow to receive cavernous surface that can provide the best adhesion of cement with metal base and, consequently, better fixation of the prosthesis to the bone.

Approaches to theoretical description and experimental investigations of wet abrasive blasting have been developed in [1]. A mathematical model, which allows to estimate the approximate linear removing material from the technological modes, has been proposed. Feasibility of treating screened and unscreened surfaces for different kinds of materials has been investigated. Simulation of the contact interaction between a single abrasive particle and a surface has been conducted [1]. The existing theories for abrasive destruction of solid bodies and technique of the technological modes estimation were summarized in [2]. A set of empirical curves for the evaluation of abrasive cutting effectiveness, quality of treated surfaces, the approximate cost of processing were given. The methods of intensifying abrasive impact were considered. Especially abrasive blasting was analyzed. Comparative characteristics of thermal abrasive blasting and sand-blasting is presented in [3]. It is ascertained the influence of technological modes on residual compressive stresses and roughness of treated surfaces. To describe relation between the technological modes of abrasive blasting and quality characteristics of the surfaces (the depth of hardened layer, the change of surface microhardness) and to take into account the physical and mechanical properties of treated surfaces, design of abrasive blasting nozzle, the characteristics of working medium (material, dimensions), shape of the surface, interaction between an abrasive jet and the surface, mathematical models were developed in [4].

The method of calculating the geometrical and technological parameters of batch-operation pneumatic abrasive blasting plant, equipped with feeding hopper for abrasive particles, was proposed.

As a rule, theoretical study of abrasive blasting by scientists from abroad are based on finite element method [5-8]. Experimental studies are usually related to the influence of treatment on physical and mechanical properties of different groups of metals and alloys. Also abrasive blasting is often used in combination with laser treatment [9, 10].

As it follows from the literature review, a promising way for theoretical study of abrasive blasting is the mathematical modeling of curvilinear surface treatment. It needs to single out and to describe analytically different shapes of the surfaces, such as spherical convex and concave surfaces, cylindrical convex and concave surfaces, elliptic and hyperbolic paraboloids. In different combinations these shapes can form a wide range of curvilinear surfaces. Further geometrical parameters of an abrasive particle trace on the curvilinear surface should be described analytically and distribution of kinetic energy will be obtained. It will allow to predict the quality characteristics (residual compressive stress distribution, microhardness, depth of hardened layer) of the curvilinear treated surfaces.

The aim of the paper is obtaining analytic dependences for assessment of the glancing impact angle to a point in the curvilinear surface of machine part for abrasive blasting.
2. Description of the approach and achieved results of own researches

A glancing impact angle of a jet is an important parameter of abrasive blasting, because it determines the nature of the abrasive particles contact with the machine’s part surface, the energy distribution of the jet at the point of contact, and hence determines the type of deformation processed material and thus affects the physical and mechanical parameters of the machine’s part surface.

The glancing impact angle of a jet to a specified point of the curvilinear surface M will be called an angle \( \alpha \) between the flight trajectory of the abrasive particle (a beam that goes from the jet pole \( O \), and passing through the given point \( M \)) and a tangent \( MB \) to the curve that describes the surface in the selected section (Fig. 1).

Using geometric consideration (Fig. 1) and take into account that \( MB \) is a tangent to the curve \( z = f(x) \) and hence
\[
\tan \beta = \frac{dz}{dx}
\]
we obtain the glancing impact angle
\[
\alpha = \frac{\pi}{2} + \arctg \frac{dz}{dx} - \arctg \frac{x}{L-z}
\]
(1)

where \( x, z \) is coordinates of the point \( M; z = f(x) \) is an equation of the curve, i.e. a line formed by a curvilinear surface section on a plane \( O,OM \) and the jet longitudinal section \( XOZ \).

Using the equation (1), we can determine the glancing impact angle \( \alpha \) of the jet to any point on the curvilinear surface.

Now we concern ourselves with the limiting cases of the machine’s part surface shape and its position relative to abrasive blasting nozzle (Fig. 2).

For the case of treating a horizontal plane (Fig. 2, a) \( \tan \beta = \frac{dz}{dx} \) and the glancing impact angle is equal:
\[
\alpha = \frac{\pi}{2} + \arctg \frac{x}{L-z}
\]

For the case of treating a vertical plane (Fig. 2, b) \( \tan \beta = \frac{dz}{dx} = \infty \) and we deal with processing a glancing jet. The impact angle is equal in such case:
\[
\alpha = \pi - \arctg \frac{x}{L-z}
\]

Now we consider the case for which the cross section of a curved surface is a circle of radius \( R \) (Fig. 3, a). This is true, if the surface has cylindrical or spherical shape.

Consider equation of the circle \( x^2 + z^2 = R^2 \) and substitute its in (1) we obtain expression that allow to calculate the glancing impact angle \( \alpha \) for cylindrical or spherical shape of the surface:
\[
\alpha = \frac{\pi}{2} + \arctg \frac{-x}{\sqrt{R^2-x^2}} - \arctg \frac{x}{L-\sqrt{R^2-x^2}}
\]
(2)
For the limiting case presented in Fig. 3, b, if the beam $OM$ is tangent to the circle, and take into consideration

that $x = \sqrt{L^2 - R^2}$ we obtain with equation (2) that the glancing impact angle $\alpha$ must be equal to zero.

Simulate changing the glancing impact angle for cylindrical or spherical surface of the machine’s part. For example, if the distance from the jet pole to the point of origin is equal to 350 mm and the radius of the surface is 150 mm, diagram of changing the glancing impact angle is shown in Fig. 4.

The presented method allows to simulate changes the glancing impact angle for freeform surface, given the appropriate function (Fig. 5).

3. Conclusions

The basic analytical expression obtained in the paper allows to calculate the value of one of the most important technological parameters of abrasive blasting that is the glancing impact angle of the abrasive jet to a specified point of the machine’s part curvilinear surface. It is an abrasive blasting parameter from which microrelief, physical and mechanical properties of the surface layers depend essentially.

Using the proposed approach it can be modeled changing the glancing impact angle for the different shapes of the machine’s part surfaces and thus changing technological modes of abrasive blasting. The obtained dependence can be used as a mathematical basis for optimization of the process and development of computer-aided systems that will allow to control abrasive blasting treatment.
References